

Gauss-Seidel for convex optimization programs

In numerical analysis, the Gauss-Seidel method belongs to the class of relaxation algorithms, also known as "coordinate descent" or "block descent" methods. Originally designed for solving linear systems, this approach has found efficiency in numerical optimization and has seen numerous applications, most recently, for instance, with the LASSO method of Robert Tibshirani in statistical regression.

The initial convergence theorems of this algorithm were established to optimize strictly convex functionals. To the best of my knowledge, the two papers presented here are the first to :

- **Relax the symmetry assumption** on the matrix in the quadratic case.
- Propose a convergence theorem when **the objective function is not strictly convex**, submitted or not to a set of linear constraints. The key point being to use in the coordinate descent, the projection onto the set of minimizers under the assumption of the lower semi-continuity of that set-valued mapping.

Bibliography

[Durand J. F. \(1972\) L'algorithme de Gauss-Seidel appliqué à un problème unilatéral non symétrique. Revue française d'automatique, informatique, recherche opérationnelle. Mathématique, tome 6, no 2, p. 23-30.](#)

[Durand J. F. \(1977\) Optimisation convexe en dimension finie par relaxation RAIRO. Analyse numérique, tome 11, no2, p. 117-134.](#)